Math 521 – Fall 26

Homework 3

Due: October 27, 2026

1. Let G be a group of order 231. Show that every Sylow 11-subgroup of G is contained in $Z(G)$.

2. Let G be a finite non-abelian group that has a subgroup of order m for each m dividing the order of G. Show that G is not simple.

3. Let G be a finite abelian group and let p, q be distinct primes. Let $K = G/G[p]$. Determine $H \leq G$ such that $K[q] = H/G[p]$.

4. Show that for any $n > 0$, the group D_{2^n} is nilpotent of class n.