Math 521 – Fall 26

Homework 2

Due: October 15, 2026

1. (a) Let G be a group and $H \leq G$. Consider the action of G on $X := G/H$ by left multiplication: $g * (kH) = gkH$ for $g, k \in G$. Show that the G_X is the largest normal subgroup of G that is contained in H.

   (b) Let G be a group that has a proper subgroup of finite index. Show that G has a proper normal subgroup of finite index.

2. Determine the centers of the groups S_n and D_n. Explain.

3. Let $H \leq G$ be finite groups such that the index $[G : H]$ is the smallest prime dividing the order of G. Show that $H \triangleleft G$.

4. (a) Let $G = \langle a, b \rangle$ be a finite group, where a, b are elements of order 2 that do not commute. Show that $G \simeq D_n$ for some $n > 2$.

   (b) Is this statement still correct if the assumption on G to be finite is removed? Explain your answer.

5. Find a subgroup of $\mathrm{GL}_2(\mathbb{C})$ of order 8, that is not abelian and is not isomorphic to D_4.