Math 521 – Fall 26

Homework 1

Due: October 8, 2026

1. For a prime p, let
$$\mathbb{Z}(p^\infty) := \left\{ e^{\frac{2\pi i m}{p^n}} : m, n \in \mathbb{N} \right\}.$$
   (a) Show that every proper subgroup of $\mathbb{Z}(p^\infty)$ is a finite cyclic group.
   (b) Let A be an abelian group containing elements $a_1, a_2, \ldots$ such that $|a_1| = p$ and $pa_{n+1} = a_n$ for all $n > 0$. Show that the subgroup of A generated by $a_1, a_2, \ldots$ is isomorphic to $\mathbb{Z}(p^\infty)$.

2. For each $n > 0$, find a subgroup of $\mathrm{GL}_2(\mathbb{C})$ that is isomorphic to D_n.

3. Let $m, n > 0$. Determine the number of group homomorphism from $\mathbb{Z}/m\mathbb{Z}$ to $\mathbb{Z}/n\mathbb{Z}$.

4. Determine all groups of order 6 up to isomorphism.

5. Let a group G act on the set of its subgroups by conjugation. For a subgroup H of G, the stabilizer of H under this action is called the *normalizer of* H and it is denoted by $N_G(H)$. Show that for any subgroup H of G, $N_G(H)$ is the largest subgroup of G in which H is normal.